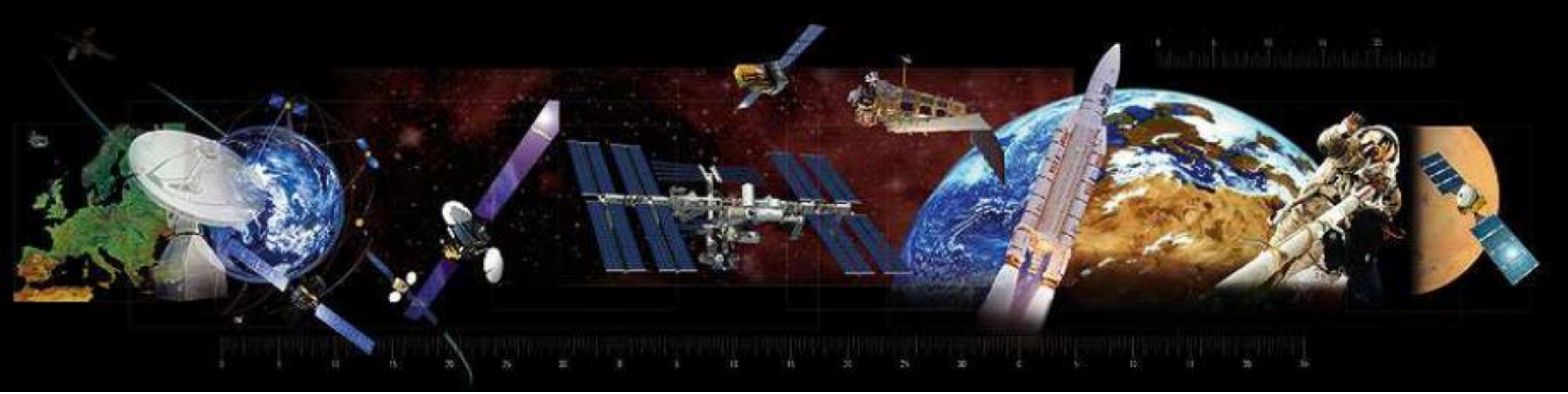

Link Budget Analysis: Digital Modulation, Part 1

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Presentation Content

Link Budget Analysis: Digital Modulation, Part 1

Refer to Atlanta RF's presentation titled: 'Link Budget – Getting Started', which can be downloaded from our website: www.AtlantaRF.com.

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Typical Sources of Communication Data

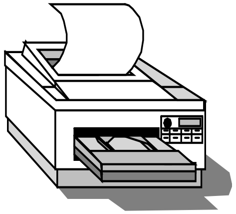
Link Budget Analysis: Digital Modulation, Part 1

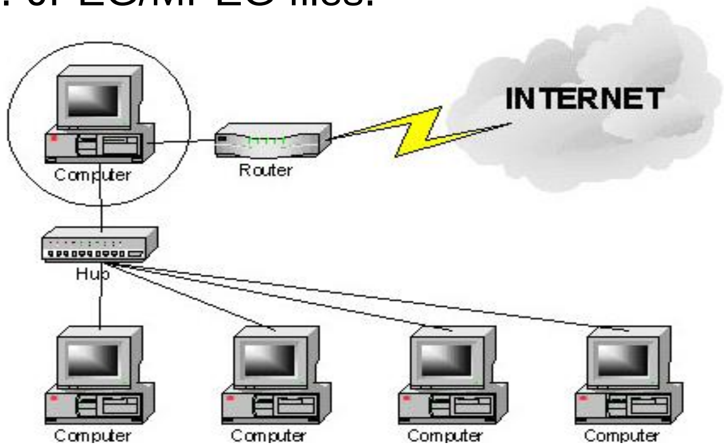
1. Analog Data Sources: Produces continuous-time output using a device that converts the real analog signal to electrical voltage.

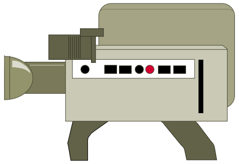
A. Speech/Voice/Telephone.
B. Music/Sound.
C. Moving and static images.
D. And also: temperature, speed, time...

2. Digital Data Sources: Produces discrete-time output using a device that processes logical digital signals (binary, ASCII).

A. Computer files/Keyboards/Monitors/Printers.
B. E-mail sent over the internet.
C. Digital storage devices (Compact Discs, DVDs, etc…)
D. JPEG/MPEG files.







Metrics for Choice of Modulation Scheme

Link Budget Analysis: Digital Modulation, Part 1

1. High spectral efficiency: η_b . . . . Signal uses a small bandwidth.
   A. Transmitted signal occupies the minimum RF channel bandwidth.
2. High power efficiency: η_p . . . . . . Detect a small signal power.
   A. Provides low bit-error rates (BER) at low Signal-to-Noise (S/N) ratios.
3. High data rates: Bits per second.
4. Robust to multipath effects & fading conditions.
5. Easy to implement and cost-effective to operate.
6. Low carrier-to-cochannel signal interference ratio.
7. Low out-of-band radiation.
8. Constant or near constant envelope:
   A. Constant envelope: Only phase is modulated; can use power-efficient non-linear amplifiers.
   B. Non-constant envelope: Phase and amplitude modulated; may need power-inefficient linear amplifiers.

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Power Efficiency of Digital Modulation

A performance metric for digital communication systems

1. Power efficiency is the ability of a modulation technique to preserve the **fidelity/quality** of digital messages at **low power levels**, and is expressed as the ratio of the signal energy per bit (E_b , watt-sec) to the noise power spectral density per bit (N_0 , watts/hertz) required to achieve a given probability of bit error rate, say BER ~ 10^{-5} :

$$\text{Power Efficiency}: \eta_p = \left[\frac{E_b}{N_0} \text{ required at the receiver input for certain } BER \right]$$

2. To obtain good fidelity/quality, the signal power usually needs to be increased for better noise immunity.
   A. Tradeoff between signal fidelity (BER) and signal power (E_b/N_o).
   B. Power efficiency describes how efficient this tradeoff is made.
3. There are cases when bandwidth is available but transmit power is limited.
   A. In these cases as 'M' goes up, the bandwidth *increases* but the required power levels to meet a specified BER remains stable.
4. Modulations that are power-limited achieve their goals with minimum expenditure of power at the expense of bandwidth. Examples are MFSK and other orthogonal signaling.

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Bandwidth Efficiency of Digital Modulation

A performance metric for digital communication systems

1. Bandwidth (Spectral) efficiency describes the ability of a modulation scheme to accommodate data within a limited frequency bandwidth. In general, it is defined as the ratio of the throughput data bit rate: R_b, in bits per second, to the required frequency bandwidth occupied by the modulated RF signal: B_T, in hertz:

$$\text{Bandwidth Efficiency}: \eta_B = \frac{R_b}{B_T}, \text{bits/second/Hz}$$

2. Bandwidth efficiency reflects how efficiently the allocated frequency bandwidth is utilized. Tradeoff between data rate: R_b and pulse width: T_s ($B_T \sim 1/T_s$).
3. Channel capacity gives an upper bound of achievable bandwidth efficiency.
4. There are situations where bandwidth is at a premium, so modulations with large throughput data rate per hertz are needed (large $\eta_B = R_b/B_T$).
5. Hence we need standards with large time-bandwidth product: $B_T T_b$.
6. The GSM standard uses Gaussian minimum shift keying (GMSK) with $B_T T_b$=0.3.
7. Modulations that achieve bit error rates at a minimum expenditure of bandwidth, but possibly at the expense of too high a signal power, are bandwidth-limited.
   A. Examples are variations of MPSK and many QAM.

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Tradeoff: BW Efficiency and Power Efficiency

Link Budget Analysis: Digital Modulation, Part 1

1. Fundamental tradeoff between Bandwidth Efficiency: η_B and Power Efficiency: η_p . . . . in general:
   A. If η_B improves, then η_p deteriorates (or vice versa).
      1) May need to waste more signal power: E_b/N_o to get a better data rate: R_b.
      2) May need to use less signal power (to save on battery life) at the expense of a lower data rate.
   B. η_B versus η_p is not the only consideration.
      1) Use other factors to evaluate → system complexity, resistance to MRC impairments, etc.
2. Adding **error control coding** improves the power efficiency (there are fewer errors), but reduces the bandwidth efficiency (redundant data bits are also transmitted, which requires more bandwidth).
3. **M-ary modulation schemes** increase the bandwidth efficiency but requires higher transmission power to keep the same bit error rate: BER.

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Digital Modulation Tradeoffs

Link Budget Analysis: Digital Modulation, Part 1

1. Linear Modulation:

A. The amplitude of the modulated transmitted signal: ***s(t)***, varies linearly with the modulating digital signal: ***m(t)***. Bandwidth efficient but power inefficient. Examples: M-ASK, M-PAM, BPSK, DPSK, QPSK, π/4 PSK, M-QAM.

B. Information encoded in carrier signal's amplitude and/or in carrier's phase.

C. Easier to adapt. More spectrally-efficient then nonlinear modulation.

D. Issues: differential encoding, pulse shaping, bit mapping.

   A. Often requires linear power amplifiers to minimize signal distortions.

2. Nonlinear Modulation:

A. The amplitude of the modulated transmitted signal: ***s(t)***, does not vary linearly with the modulating digital signal: ***m(t)***. Power efficient but bandwidth inefficient. Examples: FSK, MSK, GMSK, constant envelope modulation.

B. Information encoded in carrier signal's frequency.

C. Continuous phase (CPFSK) modulation is a special case of FM.

D. Bandwidth determined by Carson's rule[(1)] (pulse shaping).

E. More robust to channel and power amplifier's nonlinearities.

1: J.R. Carson, "Notes on the theory of modulation“, Proceedings of IRE, vol. 10, no. 1 (Feb. 1922), pp. 57-64.

Modulation: Types and Techniques

Link Budget Analysis: Digital Modulation, Part 1

1. **Analog Modulation**: When the information-bearing message signal is continuous-time analog, then the modulation is called analog modulation. Common analog modulation techniques:
   A. AM: Amplitude modulation : Message is carried in **A(t)**. $S(t) = \boldsymbol{A(t)} \cos(\omega_c t + \varphi_0)$.
   B. FM: Frequency modulation: Message is carried in **ω(t)**. $S(t) = A_0 \cos(\boldsymbol{\omega(t)} + \varphi_0)$.
   C. PM: Phase modulation : Message is carried in **φ(t)**. $S(t) = A_0 \cos(\omega_c t + \boldsymbol{\varphi(t)})$.

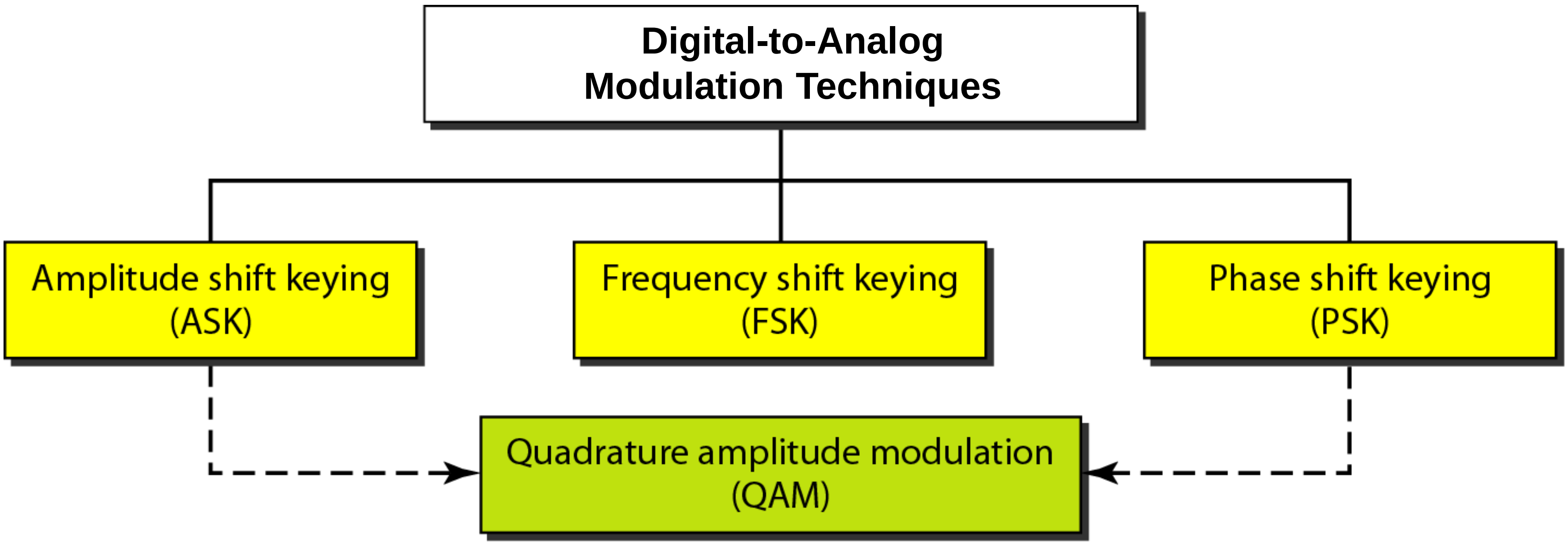
2. **Digital Modulation**: When the information-bearing message signal is discrete-time digital, then the modulation is called digital modulation. Common digital modulation techniques:
   A. ASK: Amplitude Shift Keying : Message signal changes the carrier's **amplitude**.
   B. FSK: Frequency Shift Keying: Message signal changes the carrier's **frequency**.
   C. PSK: Phase Shift Keying : Message signal changes the carrier's **phase**.
   D. QAM: Quadrature Amplitude Modulation. A combination of ASK and PSK.

3. **For Binary (2-level) Digital Modulation** (M = 2):
   A. BASK: Binary Amplitude Shift Keying.
   B. BFSK: Binary Frequency Shift Keying.
   C. BPSK: Binary Phase Shift Keying.

Types of Digital-to-Analog Modulations

$$s_i(t) = \sum_{n=-\infty}^{\infty} A_i(t)\cos(2\pi f_i t + \theta_i(t))$$

Time-varying Amplitude — Time-varying Frequency — Time-varying Phase



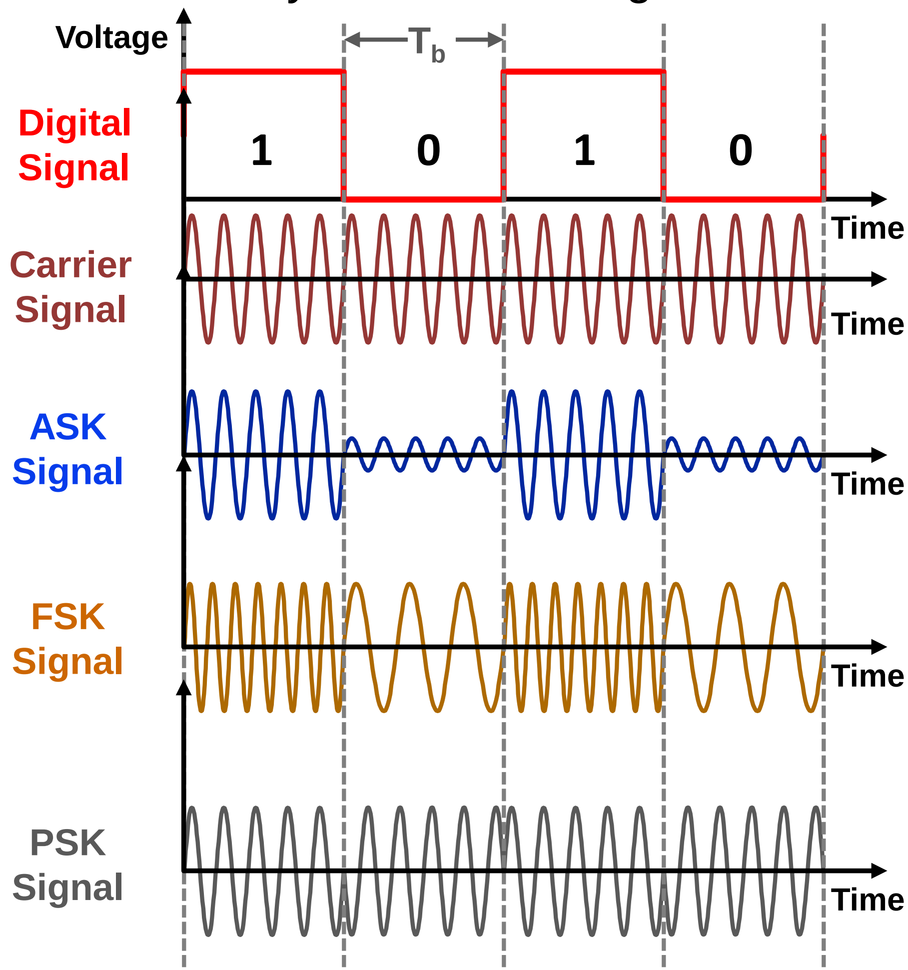
- Bit rate is the number of bits transmitted per second: $R_b = kR_s$.
- Baud rate is the number of signal elements transmitted per second: $R_s = R_b/k$.
- In the analog transmission of digital data, if a signal unit is composed of *k bits, then the bit rate is k times* higher than baud rate. Baud rate determines the channel bandwidth required to transmit the modulated signal.

Digital Bandpass Modulation Techniques

Link Budget Analysis: Digital Modulation, Part 1

In digital communications, the modulating baseband message signal: ***m(t)*** is a binary or M-ary digital data stream. The carrier is usually a sinusoidal signal.

1. **Baseband digital message signal**: ***m(t)***
2. **Analog sinusoidal carrier signal**:
   A. Carrier signal: $A_c \cos(2\pi f_c t + \phi_c)$
3. **ASK: Amplitude Shift Keying.**
   A. Message signal changes the carrier's **amplitude** : $A_i(t)$.
4. **FSK: Frequency Shift Keying.**
   A. Message signal changes the carrier's **frequency** : $f_i(t)$.
5. **PSK: Phase Shift Keying.**
   A. Message signal changes the carrier's **phase** : $\phi_i(t)$.



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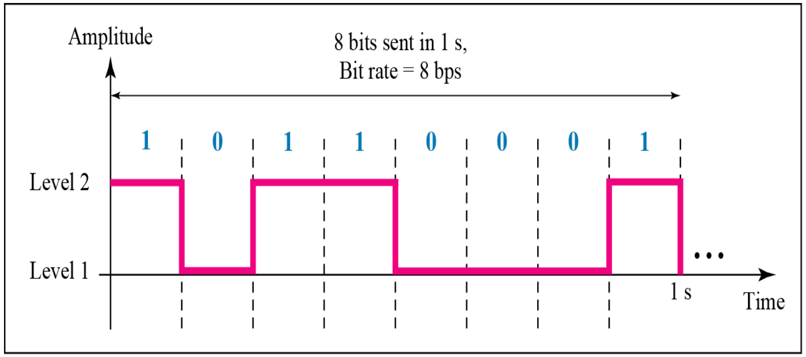
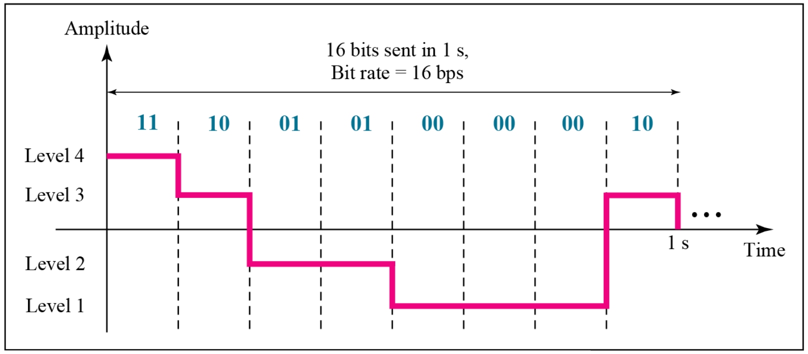
Types of Digital Modulation Techniques

Link Budget Analysis: Digital Modulation, Part 1

1. **Amplitude Shift Keying: ASK**
   A. On-Off Keying: OOK.
   B. Binary Amplitude Shift Keying: BASK.
2. **Frequency Shift Keying: FSK**
   A. Binary Frequency Shift Keying: BFSK.
   B. 4-level Frequency Shift Keying: 4-FSK.
3. **Phase Shift Keying: PSK**
   A. Binary Phase Shift Keying: BPSK.
   B. Quadrature Phase Shift Keying: QPSK, DQPSK, OQPSK, π/4-QPSK.
   C. 8-Level Phase Shift Keying: 8-PSK.
   D. 16-Level Phase Shift Keying: 16-PSK.
4. **Quadrature Amplitude Modulation: QAM**
   A. 16-QAM C. 128-QAM E. etc. . . .
   B. 64-QAM D. 265-QAM
5. **Continuous Phase Modulation: CPFSK**
   A. Minimum Shift Keying: MSK
   B. Gaussian MSK: GMSK



Digital signal with two signal levels



Digital signal with four signal levels

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Multi-level Signaling: Digital M_{ary} Modulation

Link Budget Analysis: Digital Modulation, Part 1

1. In general, multi-level (M-ary) digital communication is used to design a communication system that is more bandwidth efficient. With M-ary signaling, digital inputs with more than two modulation levels are allowed on the transmitter's input.
2. The data is transmitted in the form of symbols, each symbol is represented by k bits, so there are $M = 2^k$ different signal levels in M-ary modulation.
3. In M-ary data transmission, one of 'M' possible signals is transmitted during each signaling interval: T_s , where: $T_s = kT_b$ and T_b = bit time interval.
4. There are many different M_{ary} modulation techniques, some of these techniques modulate one carrier parameter, like: Amplitude, or Phase, or Frequency:
   - **A. M_{ary} ASK**: M_{ary} Amplitude Shift Keying: M-ASK or M-PAM.
     1) The carrier signal's **amplitude** takes on **'M' different levels**.
     2) Used in baseband transmission: Pulse Amplitude Modulation (PAM) and in bandpass transmission: ASK.
   - **B. M_{ary} PSK**: M_{ary} Phase Shift Keying: M-PSK.
     1) The carrier signal's **phase** takes on **'M' different levels**.
   - **C. M_{ary} FSK**: M_{ary} Frequency Shift Keying: M-FSK.
     1) The carrier signal's **frequency** takes on **'M' different levels**.

Modulation in Wireless Applications

Link Budget Analysis: Digital Modulation, Part 1

1. **Analog FM**: AMPS – Advanced Mobile Phone System at 850 MHz.
2. **GMSK**: Gaussian Minimum Shift Keying:
   A. GSM – Global System for Mobile Communications at 900 MHz.
   B. DCS1800 - Digital Cellular System at 1800 MHz (USA)
   C. PCS1900 – Personal Communication System at 1900 MHz (USA)
   D. DECT – Digital European Cordless Telephone at 1880 – 1900 MHz (Europe)
   E. CT2 – Cordless Telephone 2 (Canada)
3. **π/4-DQPSK**: π/4 Differential Quadrature Phase Shift Keying
   A. IS-54 at 900 MHz/IS-136 at 2 GHz (North America)
   B. PDC – Personal Digital Cellular at 800 & 1500 MHz/PHS (Japan)
4. **QPSK**(FL)/DQPSK(RL): IS-95 (North America Digital Cellular):
   A. Data Rate = 48kb/s; Bandwidth = 30kHz
   B. Bandwidth efficiency = 48/30 = 1.6bits/sec/Hz
5. **BPSK, QPSK, OFDM**: IEEE802.11 at 2.4 GHz & 5 GHz (ISM band).
6. **GFSK**: Bluetooth at 2.4 GHz (Industrial-Scientific-Medical band).

Modulation Formats in Cable

Link Budget Analysis: Digital Modulation, Part 1

Modulation	Description	Use	Comments
AM, FM, PM	Amplitude Modulation Frequency Modulation and Phase Modulation	Radio, Citizens Band, Cable	Low Spectral Efficiency.
PAL, NTSC	Phase Alternate Line, National Television System Committee	Commercial Television and Cable	Low Spectral Efficiency. Noise viewable by users.
QPSK, BPSK, FSK	Quadrature Phase Shift Keying, Binary Phase Shift Keying, Frequency Shift Keying	Cable modem return path, DVB-S, Telemetry channels	Robust in poor signal-to-noise.
VSB	Partially-suppressed -carrier Vestigial Sideband	North American broadcast digital television	Good performance in multi-path conditions.
QAM	Quadrature Amplitude Modulation	Digital cable broadcast, DVB-C, Cable modems	Requires good signal-to-noise.
S-CDMA	Synchronous Code Division Multiple Access	DOCSIS 2.0 return path	Good performance in poor signal-to-noise.

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Performance Advantages of Digital Transmission

When compared to Analog Modulation

1. Digital transmission produces **fewer data errors** than analog transmission:
   A. Data integrity & noise immunity: Easier to detect and correct information-bearing data errors, since transmitted data is binary (1's & 0's : only two distinct values) .
   B. Error coding is used to detect and correct digital transmission errors.
   C. Regenerative capability: Regenerative digital repeaters placed along the transmission channel can detect a distorted digital signal and retransmit a new, clean digital data signal. These repeaters minimize the accumulation of noise and signal distortion along the transmission channel.
2. Permits **higher transmission data rates**: Economical to build transmission links of very high bandwidth. Optical fiber designed for digital transmission.
3. **Better spectral efficiency**: Effective use of limited frequency resources (narrow bandwidth) to send a large amount of data.
4. **Security & privacy**: Enables encryption algorithms in information-bearing digital bit stream signals. Deters phone cloning and eavesdropping.
5. Easy to **multiplex multiple sources** of information: Voice, video and data in a **single** transmission channel, since all signals are made up of 1's and 0's.
6. **Easy to integrate** computer/communication systems.
7. Digital equipment consumes **less DC power** in a smaller physical size.

Disadvantages of Digital Transmission

When compared to Analog Modulation

Disadvantages:

1. More Bandwidth Needed:

A. Transmission of digitally encoded analog signals requires significantly more bandwidth than simply transmitting the original analog signal.

2. Circuit complexity:

A. Analog signals must be converted to digital pulses prior to transmission and converted back to their original analog form at the receiver: Additional encoding/decoding circuitry needed.

3. Synchronization:

A. Requires precise time synchronization between the clocks in the transmitter and in the receiver.

Digital Bandpass Modulation Process

Overview

1. Digital Modulation involves translating the baseband digital message signal: $m(t)$, to a bandpass analog signal: $s(t)$, at a carrier frequency: f_c , that is very high compared to the digital baseband frequency: f_b. The choice of carrier frequency allows placement of the composite modulated signal in a desired frequency band for signal processing. Modulation allows many signals with different carrier frequencies to share the frequency spectrum.
2. Digital Modulation is achieved by switching or ‘keying’ (i.e.: varying) the amplitude, phase and/or frequency of a high-frequency sinusoidal analog carrier signal: $s(t)$ in accordance with the incoming information-bearing digital baseband message signal: $m(t)$, a time sequence of symbols or pulses, thereby resulting in a bandpass modulated signal that is transmitted by the sender over a channel. Modulated signals propagate well through the atmosphere.
3. Changes in the amplitude, phase and/or frequency of the carrier signal are used to represent a digital state of the modulating digital baseband signal.
4. Using this technique, digital or analog data is ‘encoded’ into a digital signal.
5. A bandpass carrier signal modulated by baseband digital data has the form:

$$S_i(t) = \sum_{n=-\infty}^{\infty} A_i(t)\cos(2\pi f_i t + \theta_i(t))$$

where digital data bits are encoded in discrete time-varying amplitude A_i(t) (= *ASK*), discrete time-varying phase: θ_i(t) (= *PSK*), or discrete time-varying frequency: $\theta_i = 2\pi(f_c - f_i)t$ (= *FSK*), which remain constant over a data bit time interval: T_b .

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Basic Digital Communications System

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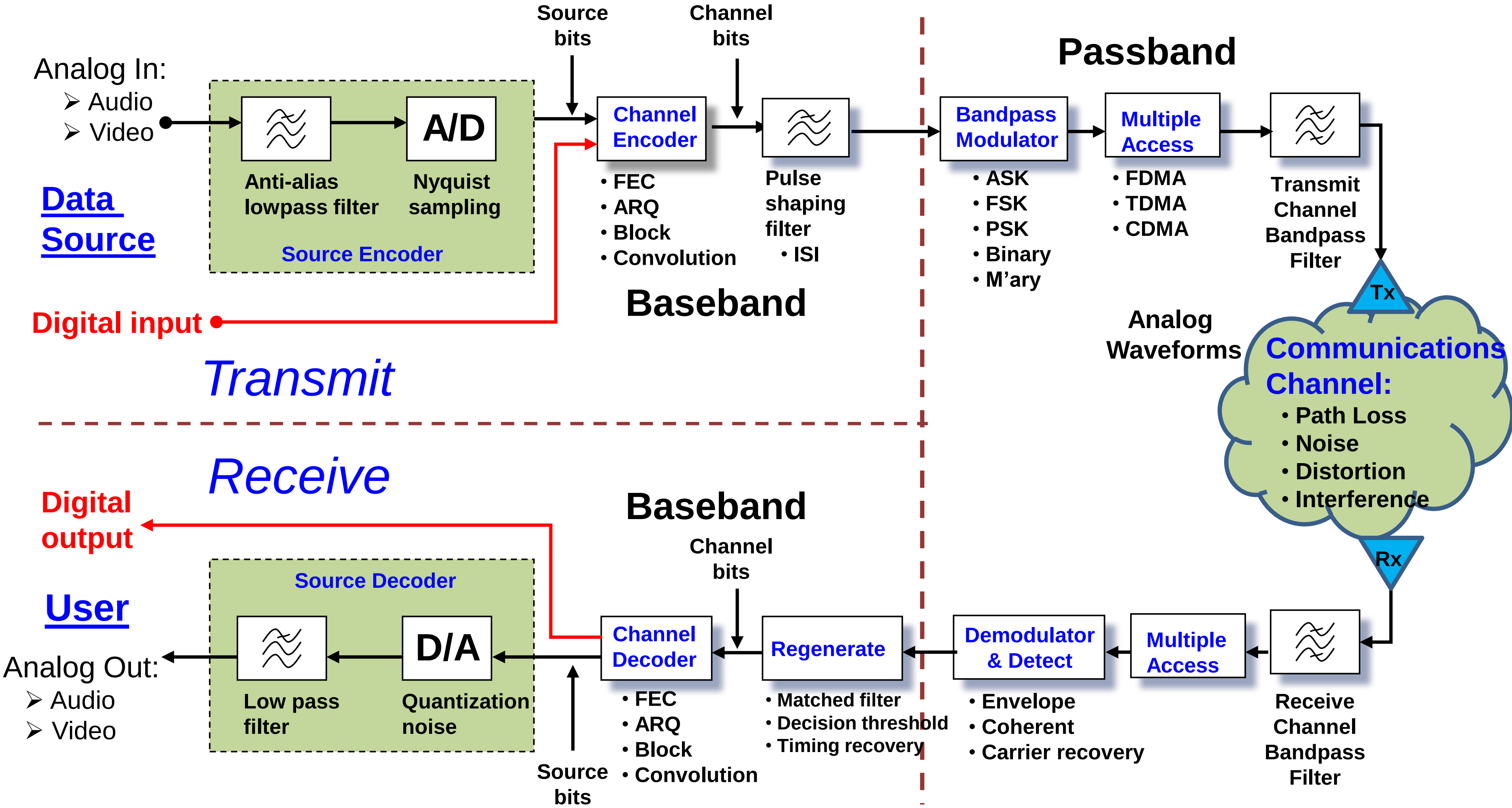

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Building a Digital Communications System

Link Budget Analysis: Digital Modulation, Part 1

1. **Source Encoder**: Samples and quantizes a time- & amplitude-varying analog signal, and converts the samples into a digital binary bit stream of 1's and 0's, then encodes it into a shorter digital signal (reduces the redundancy or reduces the bandwidth requirement: data compression). Goal: Minimize signal distortion.
2. **Channel Encoder:** Accepts the digital signal and encodes it into a longer digital signal by introducing redundant data bits in the information sequence for the purpose of combating the effects of noise and interference in the transmission channel, thereby minimizing transmission errors.
3. **Modulator:** Converts the digital information data sequences into high frequency carrier waveforms that are compatible with the characteristics of the transmission channel. Varies the amplitude, phase and/or frequency of the carrier waveform.
4. **Transmission:** Carrier modulated digital symbols are transmitted towards the desired destination, using a certain physical medium (Guided: cable, optical fiber and Unguided: wireless).
5. **Channel estimation:** Generally, transmission channels may introduce distortion to the source signal, and the characteristics of the channel distortion need to be estimated or identified at the receiver end, in order to reduce or eliminate the distortion and recover the original signal. This is called channel estimation or identification.

Bit Error Rate (BER): An Introduction
Link Budget Analysis: Digital Modulation, Part 1

1. Bit Error Rate is a major indicator of the health of the communication system.
2. As data is transmitted some of the bits may not be received correctly.
3. The more bits that are incorrect, the more the signal will be affected.
4. It's important to know what portion of the bits are in error.
5. Need to know how much margin the system has before failure.
6. Good signal: BER < 10^{-10} .
7. Threshold for visible degradation: BER ~ 10^{-6} .
8. Example:
   A. A 256QAM channel transmits at a symbol rate of 5M symbols per second.
   B. Bit rate = 8 bits per symbol X 5M symbol per second = 40M bits per second.
   C. Error Incident = Bit rate X BER = Errors Per Second.

BER	Error Frequency	Error Incident
10^{-12}	1 in 1 Trillion bits	25000 secs between errs (6.94 hrs)
10^{-11}	1 in 100 Billion bits	2500 secs between errs (41.67 mins)
10^{-10}	1 in 10 Billion bits	250 secs between errs (4.167 mins)
10^{-9}	1 in 1 Billion bits	25 seconds between errors
10^{-8}	1 in 100 Million bits	2.5 seconds between errors
10^{-7}	1 in 10 Million bits	4 errors per second
10^{-6}	1 in 1 Million bits	40 errors per second
10^{-5}	1 in 100 Thousand bits	400 errors per second
10^{-4}	1 in 10 Thousand Bits	4000 errors per second
10^{-3}	1 in 1 Thousand bits	40000 errors per second

Bit Rates of Digital Transmission Systems

Link Budget Analysis: Digital Modulation, Part 1

System	Bit Rate	Observations
Telephone twisted pair	33.6-56 kbps	4 kHz telephone channel
Ethernet twisted pair	10 Mbps, 100 Mbps	100 meters of unshielded twisted copper wire pair
Cable modem	500 kbps-4 Mbps	Shared CATV return channel
ADSL twisted pair	64-640 kbps in, 1.536-6.144 Mbps out	Coexists with analog telephone signal
2.4 GHz radio	2-11 Mbps	IEEE 802.11 wireless LAN
28 GHz radio	1.5-45 Mbps	5 km multi-point radio
Optical fiber	2.5-10 Gbps	1 wavelength
Optical fiber	>1600 Gbps	Many wavelengths

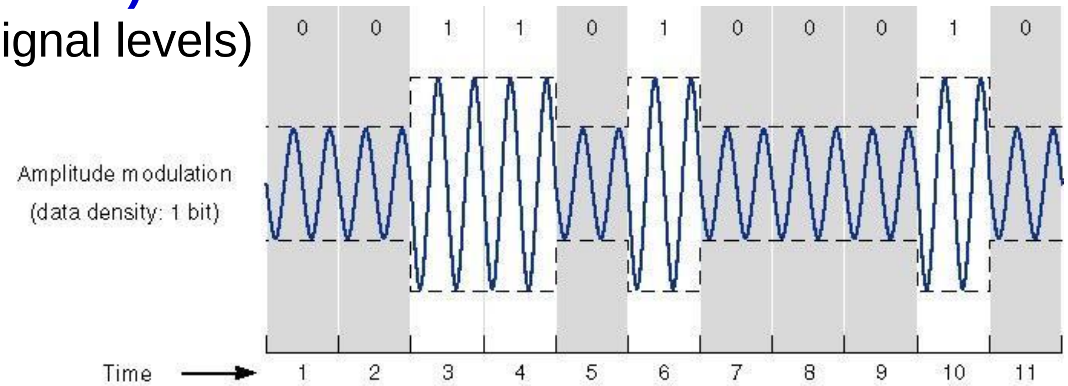
Examples of Transmission Channels

Channel	Bandwidth	Bit Rates
Telephone voice channel	3 kHz	33 kbps
Copper pair	1 MHz	1-6 Mbps
Coaxial cable	500 MHz (6 MHz channels)	30 Mbps/ channel
5 GHz radio (IEEE 802.11)	300 MHz (11 channels)	54 Mbps / channel
Optical fiber	Many TeraHertz	40 Gbps / wavelength

ASK: Amplitude Shift Keying

One dimensional linear modulation

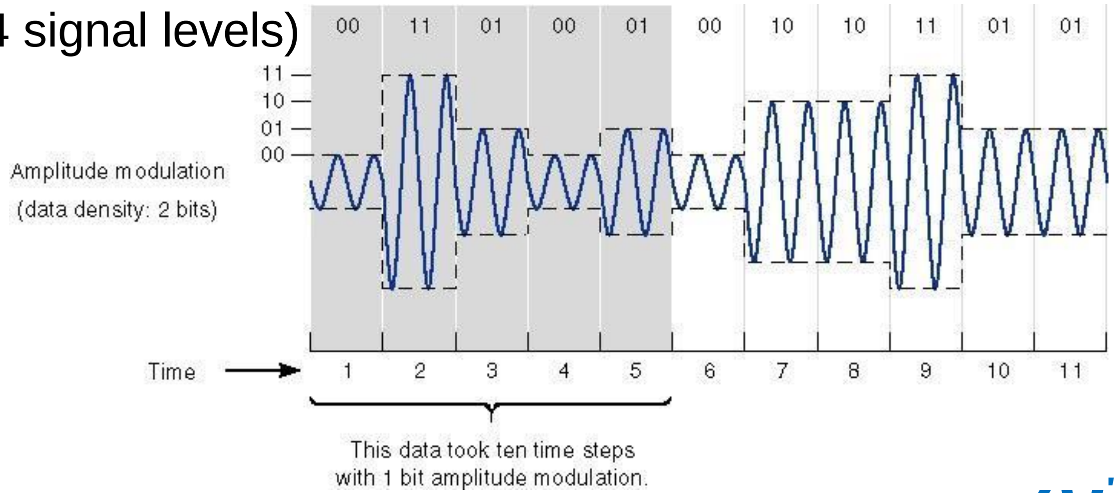
1. **Binary (2-level) ASK: BASK**
   (1-bit: 2 signal levels)



Binary Data

BASK signal

2. **4-level ASK: 4-ASK**
   (2-bits: 4 signal levels)



Binary Data

4-ASK signal

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ASK: Amplitude-Shift Keying

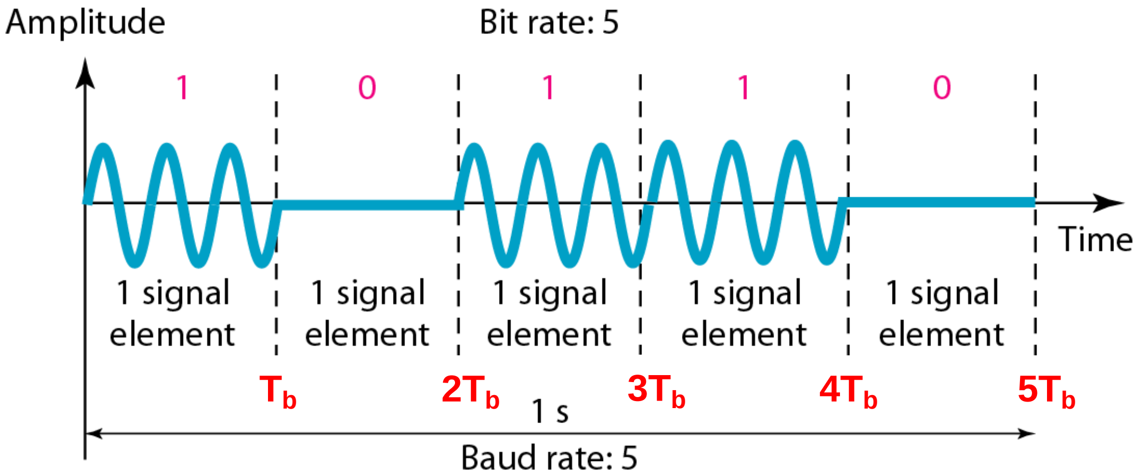
Basis of operation

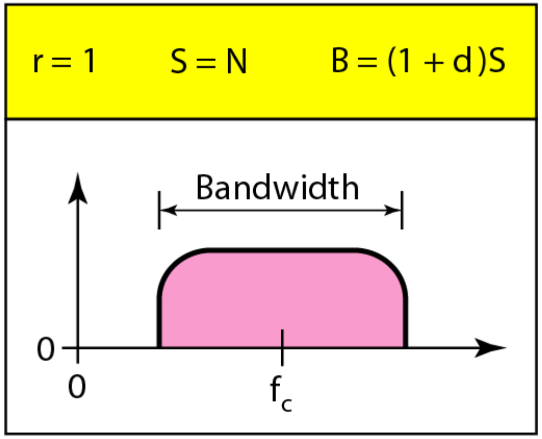
1. When the baseband signal modulates the amplitude of the carrier signal, the process is called "amplitude modulation". For digital baseband signals, it is called "Amplitude Shift Keying" : ASK . Also referred to as PAM: Pulse Amplitude Modulation.
2. Amplitude-Shift Keying (ASK) is a form of digital modulation that represents digital data solely as variations in the amplitude of a carrier signal.
3. In ASK, the amplitude of the carrier signal is changed between two (or more) levels by the digital information message signal: ***m(t)*** to represent a binary bit '0' or a binary '1' . The carrier signal's center frequency: f_c and phase: φ_c remain constant.
4. For binary ASK (BASK) , binary digit '1' is represented by the presence of the carrier signal, at constant amplitude, during a bit period: T_b , while binary bit '0' is the absence of the carrier during a bit period. If T_b indicates the time duration of one information bit, the two time-limited modulated signals can be expressed as:

$$S(t)_{ASK} = A_c\, m(t) \cos\left(2\pi f_c t + \phi_c\right) = \begin{cases} \sqrt{\dfrac{2E_i(t)}{T_b}} \cos(2\pi f_c t + \varphi_c) & \text{for a binary 1 ; } m(t) = 1 \\ 0 & \text{for a binary 0 ; } m(t) = 0 \end{cases}$$

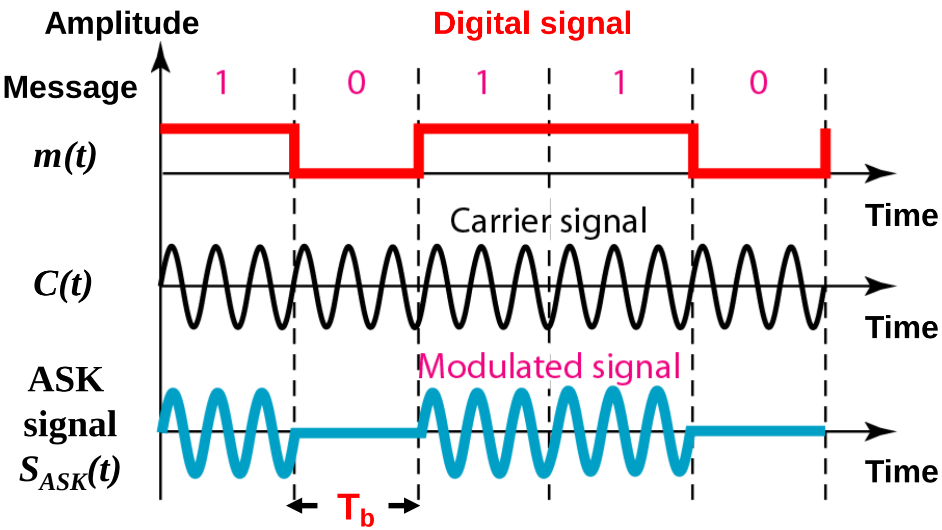
5. Carrier frequency: $f_c = n_c / T_b$, Hertz, for some fixed integer: n_c .
6. On-Off Keying (OOK) is also called Amplitude Shift Keying (ASK), which consists of keying (i.e.: switching) a carrier sinusoid on and off with a uni-polar binary signal.
7. Since noise affects the amplitude of a signal, ASK is highly susceptible to noise interference, fading, and electromagnetic induction. ASK is also most susceptible to the effects of non-linear devices, which compress and distort the signal's amplitude. It is rarely used on its own.

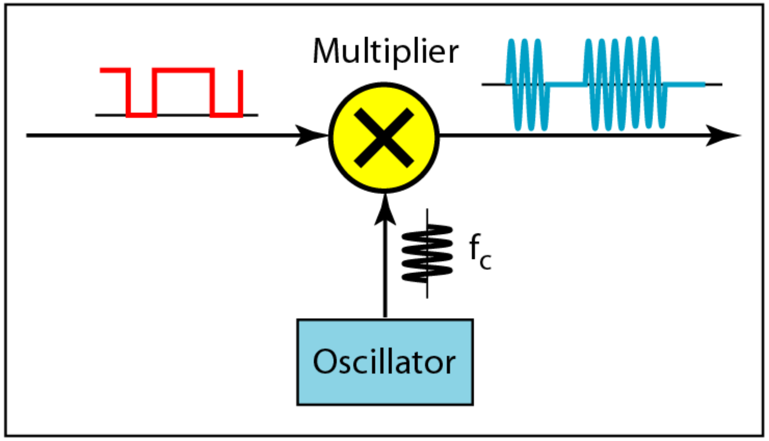
BASK: Binary Amplitude Shift Keying





Implementation of Binary ASK:





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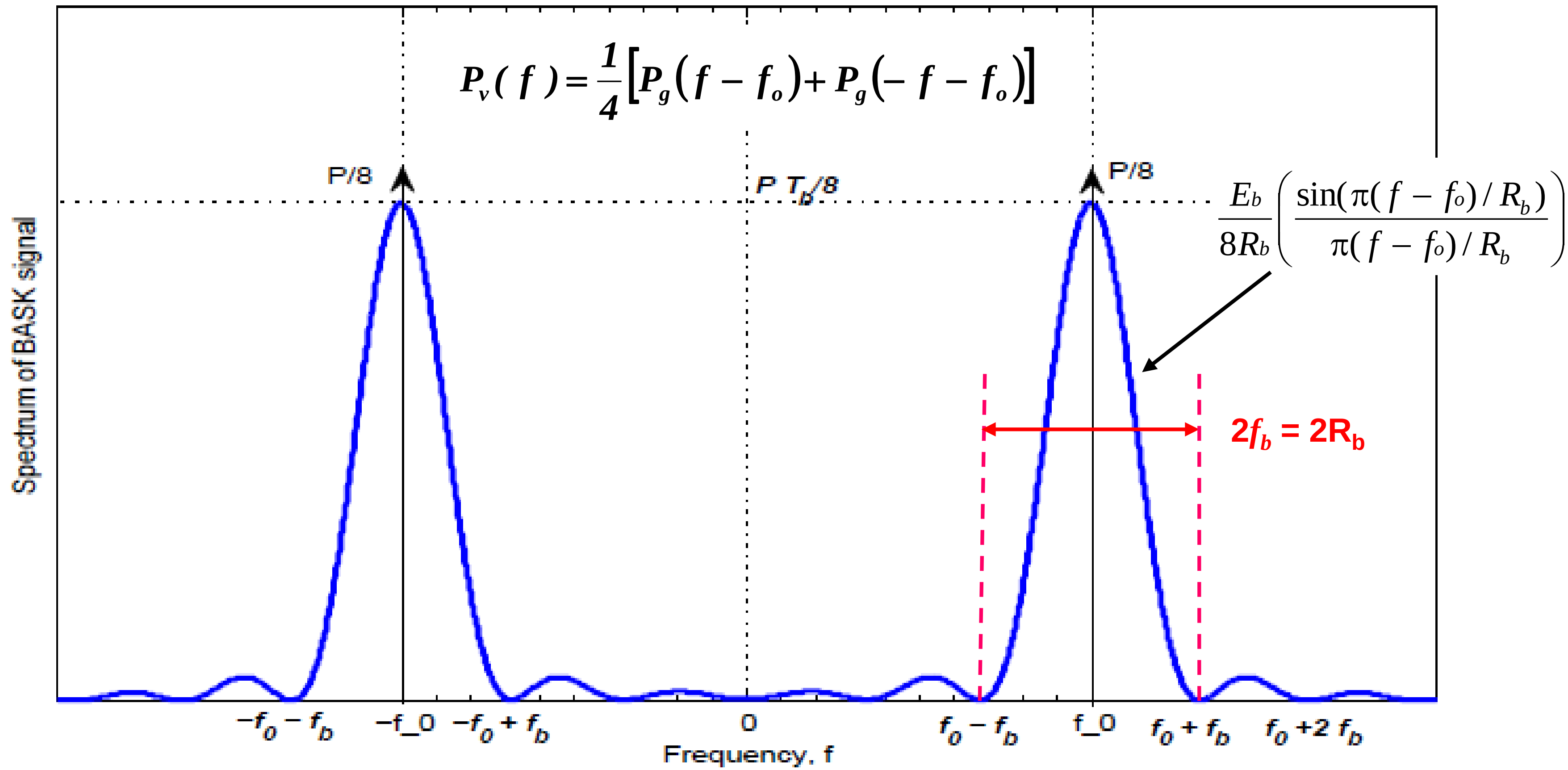
Power Spectral Density of Bandpass Binary ASK

Assumes baseband rectangular pulses:



- Energy per bit: $E_b = P\,T_b$, watts-second.
- Null-to-Null RF transmission bandwidth: $B_{null} = (f_o + f_b) - (f_o - f_b) = 2f_b = 2R_b = 2/T_b$.
- Bandwidth with 95% of the total transmitted power: $B_{95\%} = 3f_b$ (Hz), centered at f_o .

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Error Probability for M-$_{\text{ary}}$ Amplitude Shift Keying

In an Additive White Gaussian Noise (AWGN) channel

A. Modulated signal for Multi-Level ASK modulation :

$$S(t)_{ASK} = \sqrt{\frac{2E_i(t)}{T_b}}\cos(2\pi f_c t) = \sqrt{\frac{6(2i-1-M)^2 E_s}{(M^2-1)T_b}}\cos(2\pi f_c t),\ where\ E_i = E_g(2i-1-M)^2$$

B. Probability of symbol error for coherently detected Multi-Level ASK modulation :

$$P_{se,MASK} = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{2E_g}{N_o}}\right) = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{(6\log_2 M)E_b}{(M^2-1)N_o}}\right) = \frac{(M-1)}{M}\, erfc\left(\sqrt{\frac{(3\log_2 M)E_b}{(M^2-1)N_o}}\right)$$

$$where:\ E_s = (\log_2 M)E_b = \frac{(M^2-1)}{3}E_g = Average\ energy/symbol.$$

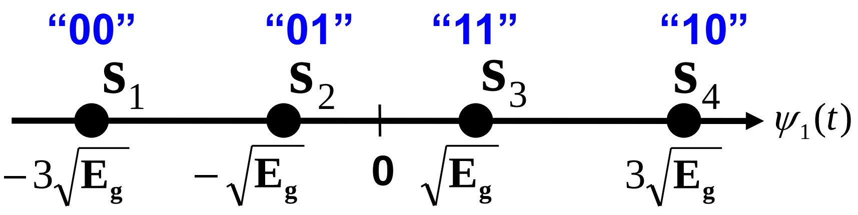
C. Probability of bit error (BER) for M-ary ASK :

$$P_{be,MASK} = \frac{P_{se,MASK}}{\log_2 M} = \frac{P_{se,MASK}}{k},\ where:\ k = \log_2 M$$

D. Binary ASK (M = 2) bit error probability :

$$P_{be,BASK} = Q\left(\sqrt{\frac{E_s}{N_o}}\right) = Q\left(\sqrt{\frac{E_b}{N_o}}\right) = \frac{1}{2}\, erfc\left(\sqrt{\frac{E_b}{2N_o}}\right)$$

4-ASK Signal Constellation Diagram:



Error Probability for M-$_{ary}$ Amplitude Shift Keying

In an Additive White Gaussian Noise (AWGN) channel

1. The average probability of bit error (BER) for Multi-Level Amplitude Shift Keying (M-ASK) modulation using coherent detection is:

$$P_{be,MASK} = \frac{2(M-1)}{M\log_2 M} Q\left(\sqrt{\frac{2E_g}{N_o}}\right) = \frac{2(M-1)}{M\log_2 M} Q\left(\sqrt{\frac{(6\log_2 M)E_b}{(M^2-1)N_o}}\right) = \frac{(M-1)}{M\log_2 M} erfc\left(\sqrt{\frac{(3\log_2 M)E_b}{(M^2-1)N_o}}\right)$$

$$\text{where: } E_s = (\log_2 M)E_b = \frac{(M^2-1)}{3}E_g = \text{Average energy/symbol and } k = \log_2 M \text{, bits / symbol.}$$

M	*k*	Probability of Bit Error (BER): P_{be}	M	*k*	Probability of Bit Error (BER): P_{be}
4	2	$P_{be,4\,ASK} = \frac{3}{8} erfc\left(\sqrt{\frac{6E_b}{15N_o}}\right)$	8	3	$P_{be,8\,ASK} = \frac{7}{24} erfc\left(\sqrt{\frac{9E_b}{63N_o}}\right)$
16	4	$P_{be,16\,ASK} = \frac{15}{64} erfc\left(\sqrt{\frac{12E_b}{255N_o}}\right)$	32	5	$P_{be,32\,ASK} = \frac{31}{160} erfc\left(\sqrt{\frac{15E_b}{1{,}023N_o}}\right)$
64	6	$P_{be,64\,ASK} = \frac{63}{384} erfc\left(\sqrt{\frac{18E_b}{4{,}095N_o}}\right)$	128	7	$P_{be,128\,ASK} = \frac{127}{896} erfc\left(\sqrt{\frac{21E_b}{16{,}383N_o}}\right)$
256	8	$P_{be,256\,ASK} = \frac{255}{2048} erfc\left(\sqrt{\frac{24E_b}{65{,}535N_o}}\right)$	512	9	$P_{be,512\,ASK} = \frac{511}{4608} erfc\left(\sqrt{\frac{27E_b}{262{,}143N_o}}\right)$

Probability of Bit Error (BER): M_{ary} ASK

In an Additive White Gaussian Noise (AWGN) channel

Probability of symbol error for coherently detected Multi-Level ASK modulation :

$$P_{se,MASK} = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{2E_g}{N_o}}\right) = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{(6\log_2 M)E_b}{(M^2-1)N_o}}\right) = \frac{(M-1)}{M} erfc\left(\sqrt{\frac{(3\log_2 M)E_b}{(M^2-1)N_o}}\right)$$

$$where:\ E_s = (\log_2 M)E_b = \frac{(M^2-1)}{3} E_g = Average\ energy/symbol\ and\ k = \log_2 M,\ bits/symbol.$$

k,bits/ symbol =		1	2	3	4	5
M signal levels =		2	4	8	16	32
E_b/N_o, dB	E_b/N_o	**Probability of Bit Error: $P_{be,MASK}$**				
0	1.000	0.07865	0.13916	0.17295	0.17789	0.1674
2	1.585	0.03751	0.09756	0.14612	0.16391	0.1607
4	2.512	0.0125	0.05862	0.11576	0.14691	0.1523
6	3.981	0.00239	0.02787	0.08347	0.12667	0.1419
8	6.310	1.91E-04	0.00925	0.05232	0.10334	0.1292
10	10.000	3.87E-06	0.00175	0.02653	0.07781	0.114
12	15.849	9.01E-09	1.39E-04	0.00972	0.05202	0.096
14	25.119	6.81E-13	2.76E-06	0.00215	0.0291	0.0757
16	39.811	0.0E+00	6.25E-09	2.17E-04	0.0124	0.0542
18	63.096	0.0E+00	4.52E-13	6.35E-06	0.00347	0.0337
20	100.00	0.0E+00	0.0E+00	2.63E-08	5.05E-04	0.0168
22	158.49	0.0E+00	0.0E+00	4.97E-12	2.63E-05	0.006
24	251.19	0.0E+00	0.0E+00	0.0E+00	2.72E-07	0.0013

Probability of bit error (BER) for Multi-Level ASK :

$$P_{be,MASK} = \frac{P_{se,MASK}}{\log_2 M} = \frac{P_{se,MASK}}{k}$$

$where:\ k = \log_2 M$, bits/symbol

The complementary error function: '*erfc*' is built into most spreadsheet software programs, like: Excel.

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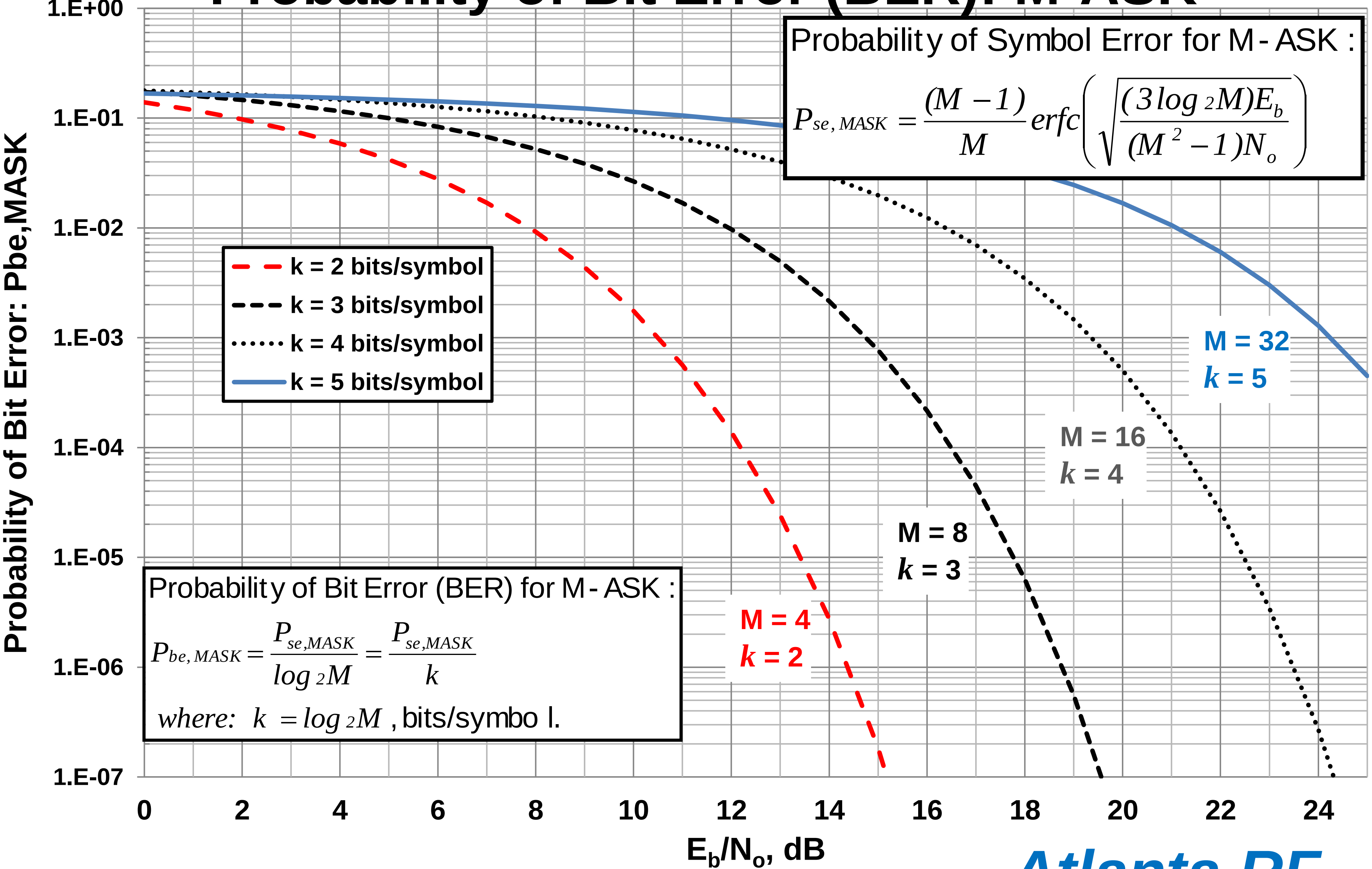
Probability of Bit Error (BER): M-ASK



Probability of Symbol Error for M-ASK:

$$P_{se,\,MASK} = \frac{(M-1)}{M}\,erfc\left(\sqrt{\frac{(3\log_2 M)E_b}{(M^2-1)N_o}}\right)$$

Probability of Bit Error (BER) for M-ASK:

$$P_{be,\,MASK} = \frac{P_{se,MASK}}{\log_2 M} = \frac{P_{se,MASK}}{k}$$

where: $k = \log_2 M$, bits/symbol.

E_b/N_o = Signal energy per bit over Noise density per bit

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Summary: Digital Modulation, Part 1

1. Digital Modulation continues to dominate the world of data & voice communication with high throughput within a congested frequency spectrum at affordable cost.
2. Design trade-offs for power-limited systems and bandwidth-limited systems often narrows the choice of digital modulation techniques.
3. Amplitude Shift Keying provides a simple & cost-effective method for communication, but is rarely used on its own, due to poor susceptibility to noise and distortion.
4. Look for additional presentations from *Atlanta RF* on Digital Modulation techniques, and visit our website: www.AtlantaRF.com to download these and other topics on Link Budget Analysis.

Refer to background material in Atlanta RF's presentation titled: 'Link Budget – Getting Started' , which can be downloaded from our website: www.AtlantaRF.com.

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Atlanta RF LLC was founded to provide engineering solutions, design software solutions, and product development solutions to the high-frequency RF/microwave industry in the areas of: Telecommunications (ground segment), Satellite (space segment) and military/defense (RF front-ends).

Through teamwork, Atlanta RF applies our diverse technical experience to your project's challenges with creative and innovative solutions while holding ourselves accountable fo the results. With professionalism and commitment to our clients, Atlanta RF will be there for you, both today and tomorrow.

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- Atlanta RF ***Software*** : **Sales@AtlantaRF.com**
- Atlanta RF ***Designs*** : **Designs@AtlantaRF.com**

Or, contact Atlanta RF by phone at: 678-445-5544, to reach our Atlanta-area office in Georgia, USA, and discuss our support to your current or future projects & products.

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